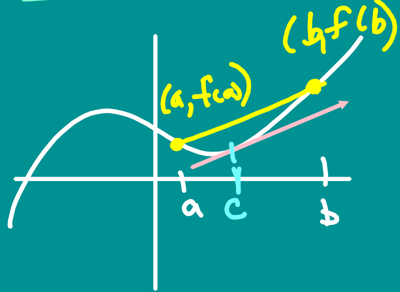
CURVE SKETCHING

Mean Value Theorem for Derivatives



1) f is continuous on $[a,b]$
2) f is diffrentiable on (a,b)

3) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$f(x) = x^3 - 3x^2 + 2x \quad [0,2]$

$f'(x) = 3x^2 - 6x + 2$

$3c^2 - 6c + 2 = \dfrac{f(2)-f(0)}{2-0}$ $\qquad f(2) = 8-12+4$
$\qquad = 0$

$3c^2 - 6c + 2 = \dfrac{0-0}{2}$

$3c^2 - 6c + 2 = 0$

$\cancel{(3c\quad)(c\quad)} = 0$

$c = \dfrac{6 \pm \sqrt{36-4(3)(2)}}{2(3)}$

$= \dfrac{6 \pm \sqrt{12}}{6}$

$= \dfrac{6 \pm 2\sqrt{3}}{6}$

$= \dfrac{3 \pm \sqrt{3}}{3}$ $\qquad \dfrac{3+1.732}{3} = \dfrac{4.7}{3} \simeq 1.6$

$\dfrac{3-1.7}{3} = \dfrac{1.3}{3} = 0.4$

$f(x) = x^4 - 4x^3 + 10$

$f'(x) = 4x^3 - 12x^2 = 0$

$\Rightarrow \underline{4x^2}(x-3) = 0$

$4x^2 = 0 \qquad x = 3$

$x^2 = 0$

$x = 0$

critical points

f' M = $+\cdot-$ $\quad$ $+\cdot-$ $\quad$ $+\cdot+$

$- \quad | \quad - \quad | \quad +$

$-1 \quad 0 \quad 1 \quad 3 \quad 4$

Concavity

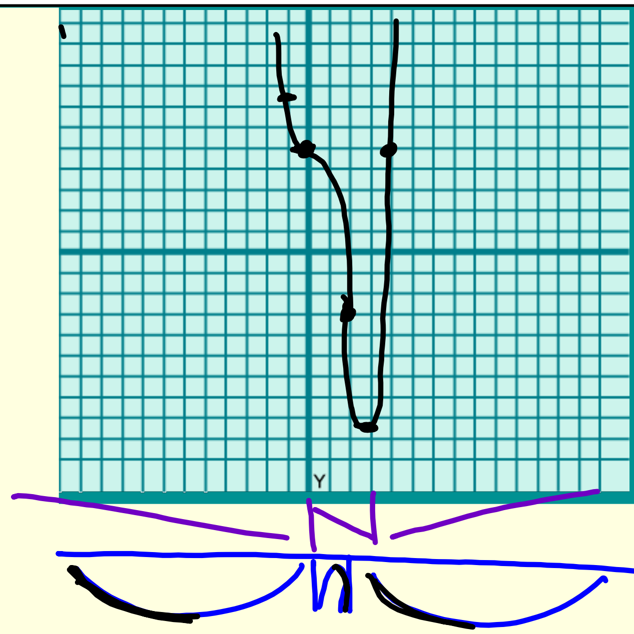
$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$x = 0 \quad x = 2$

$+ \quad | \quad \overset{+\cdot-}{-} \quad | \quad +$

$0 \quad 1 \quad 2$



x	f(x)
0	10
3	-17
2	-6
-1	15
4	10

Critical pt. — Point of non-differentiability
Where f' is undefined, but f is defined.

$f(x) = 3x^{2/3} - x$

$f'(x) = 2x^{-1/3} - 1$

$\Rightarrow \dfrac{2}{\sqrt[3]{x}} - 1 = 0$

$\dfrac{2}{\sqrt[3]{x}} = 1$

$(2)^3 = (\sqrt[3]{x})^3$

$8 = x \qquad x = 0$

pt of non-diff.

f': $\frac{2}{-1} - 1$, $\frac{2}{1} - 1$, $\frac{2}{3} - 1$

$-$ | $+$ | $-$

$-1 \quad 0 \quad 1 \quad 8 \quad 27$

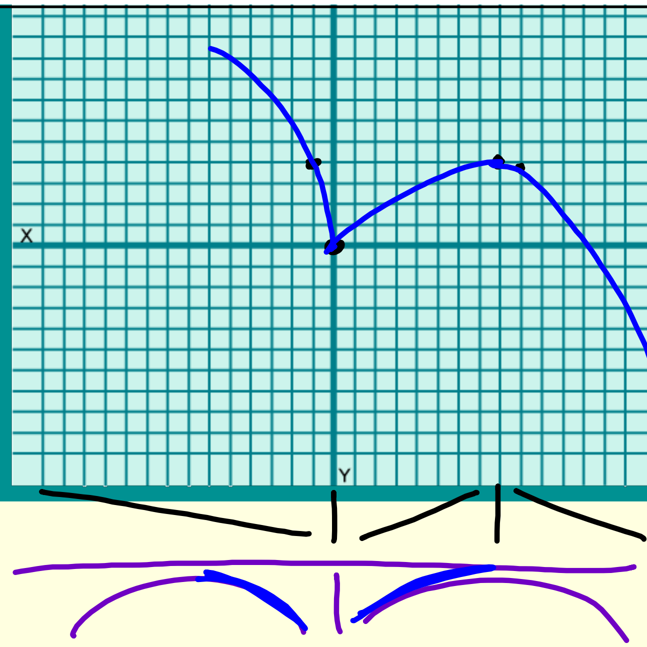
$f''(x) = -\frac{2}{3}x^{-4/3}$

$\Rightarrow \dfrac{-2}{3\sqrt[3]{x^4}} = 0$

$x = 0$

$-$ | $-$

$-1 \quad 0 \quad 1$



0	0
8	4
-1	4
9	3.98