Asymptotes

Vertical

$\lim_{x \to \#} f(x) = \pm\infty$

↑ where denom = 0

Horizontal

$\lim_{x \to \pm\infty} f(x) = \#$

Curvilinear

- the numerator is 2 or more powers higher than the denom.

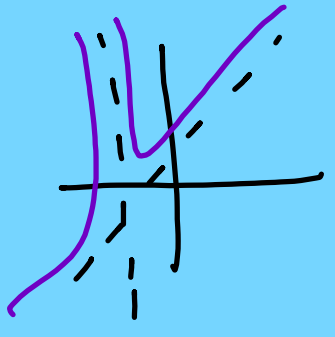
Slant (oblique)

- numerator is one power higher than the denominator
- Use long division to find

$$f(x) = \frac{4x^3 - 3x^2 - 5}{2x^2 + x + 3}$$

$$\begin{array}{r} 2x - 5/2 \\ 2x^2 + x + 3 \overline{)\,4x^3 - 3x^2 + 0x - 5} \\ \underline{-4x^3 \mp 2x^2 \mp 6x} \\ -5x^2 - 6x - 5 \end{array}$$

$y = 2x - 5/2$



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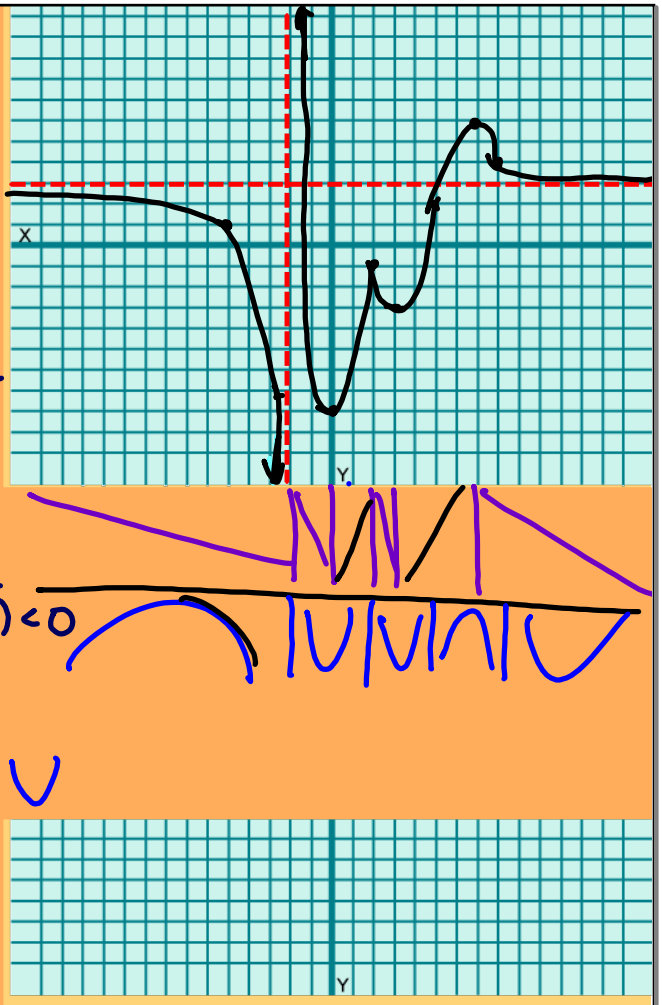
$\lim_{x \to +\infty} f(x) = 3$ ← Horz. $y = 3$

$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = +\infty$ } Vertical $x = -2$

$f(-5) = 1 \quad f(0) = -8 \quad f(2) = -1$
$f(3) = -3 \quad f(5) = 2 \quad f(7) = 6$
$f(8) = 4$

$(-\infty, -2)\ (-2, 0)\ (2, 3)\ (7, \infty)\ f'(x) < 0$
$(0, 2)\ (3, 7)\ f'(x) > 0$
$(-\infty, -2)\ (5, 8)\ f''(x) < 0$
$(-2, 2)\ (2, 5)\ (8, \infty)\ f''(x) > 0$



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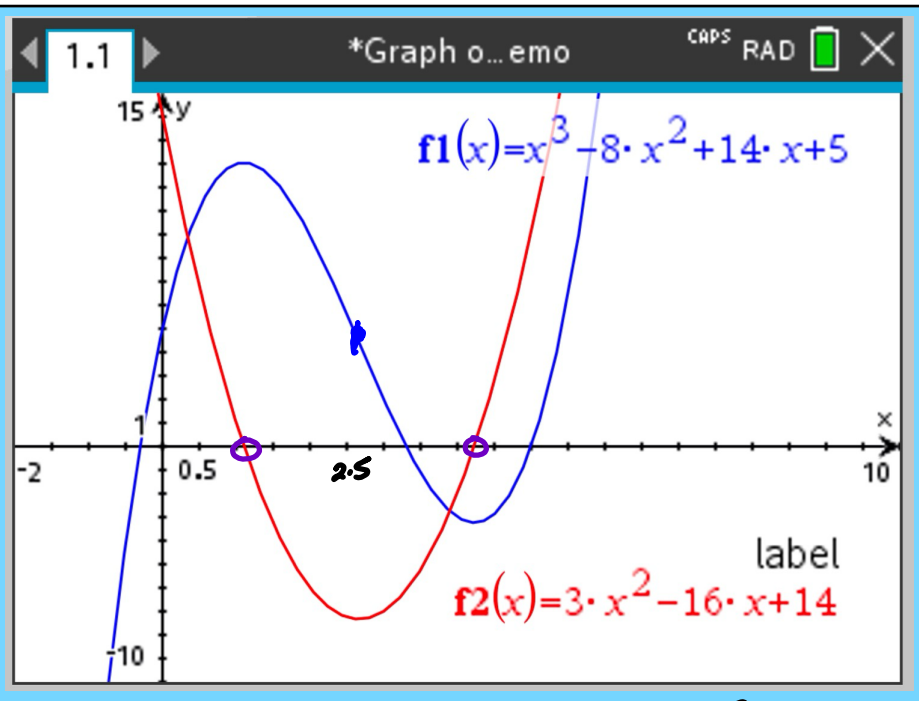

Crit pts
Where $f'(x)=0$

+ − +
1 4.5

Where $f'(x) \geq 0$ or ≤ 0
above or below x-axis

Rel Max/Min
Mountain Test

Infl Pts = rel extrema of the f'

− +
a.s

Concavity −
Where f' is inc/dec

Crit Pts. $f'(x)=0$

Inc $(-6,1)$ $(7,\infty)$

Dec. $(-\infty,-6)$ $(1,7)$

Rel Max/Min

Concavity

Infl pts.

Concave up $(-\infty,-4)(3,\infty$

Concave down $(-4,3)$