November 10, 2025

Combinatorics & Probability

How many ways are there to perform an action.

Permutations - # of ways a group of objects can be arranged in different patterns

Combinations - how many groups can be selected from a given set of objects

Linear Permutations

1) Arrange all objects
THUNDER $= n!$

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

2) Arrange a small group chosen from a larger group

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

total, use

8 cars = How many ways can 5 of 8 cars park in 5 adjacent parking stalls?

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$

$$ {}_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} $$

$$ \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}} $$

3) Alike Objects
MISSISSIPPI

$$ \frac{11!}{4!\,4!\,2!} $$

total! / alike! alike!

S I P

look same
identical
indistinguishable

4) Repeated objects OR Specific locations

Draw blanks.

Radio station call letters
- must start with K or W

$2 \cdot 26 \cdot 26 \cdot 26$
$= 35{,}152$

COMBINATIONS - Select groups of objects

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

$${}_9C_2 = \frac{9!}{7!\,2!}$$

$$= \frac{9 \cdot \cancel{8}^{4} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{2} \cdot 1}$$

$$= 36$$

How many ways can a committee of 2 guys + 2 gals be chosen from a group 6 guys + 5 gals?

(guys groups) · (girls groups)

$${}_6C_2 \cdot {}_5C_2$$

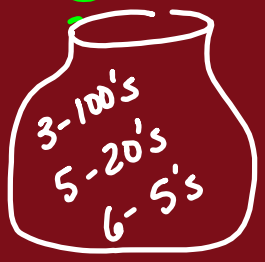
$$15 \cdot 10$$

$$= 150$$

PROBABILITY = $\frac{\text{ways to succeed}}{\text{total possible outcomes}}$ ODDS = $\frac{\text{ways to succeed}}{\text{ways to fail}}$

Prob (Hey Dude's) = $\frac{1}{6}$

Odds (not wearing Hey Dudes) = $\frac{5}{1}$



Pick 3.

Prob (3 100's) = $\frac{{}_3C_3}{{}_{14}C_3} = \frac{1}{364}$

Prob (1 100 & 2 20's) = $\frac{{}_3C_1 \cdot {}_5C_2}{{}_{14}C_3}$ AND = Multiply

$= \frac{3 \cdot 10}{364} = \frac{30}{364} = \frac{15}{182}$

Always find probability first!

Prob = $\frac{15}{182}$ $\frac{\text{suc}}{\text{total}}$

Odds = $\frac{\text{suc}}{\text{fail}} = \frac{15}{167}$

↑ 182-15

38 DVDs – 4 defective, Select 3.

Prob (at least one good DVD) =

0 { 1 good + 2 def OR 2 good + 1 def OR 3 good

P(at least one) = 1 − Prob(none)

= 1 − Prob(no good)

$= 1 - \frac{{}_4C_3}{{}_{38}C_3} = \frac{2108}{2109}$