December 2, 2025

SEMESTER REVIEW

$\lim_{x\to\infty} e^x = +\infty$ $\quad \lim_{x\to\infty} \ln x = +\infty$

$\lim_{x\to-\infty} e^x = 0$ $\quad \lim_{x\to0^+} \ln x = -\infty$

$0\cdot\infty$

$\infty-\infty$

1^∞

$0^0,\ \infty^0$

LIMITS

- $x\to\#$
  - Sub in #
    - $0/0$: factor, conjugate, L'Hop
    - $\#/0$: Check one-sided limits for $\pm\infty$
- $x\to\pm\infty$
  - poly: Use highest power term
  - rational: Use highest power term in top + bottom. If $\frac{\infty}{\infty}$, Use L'Hop
- Special
  - trig, e^x, $\ln x$: Visualize graph
  - Indet. forms: Convert to fraction, Use L'Hops

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2-2x+1}}{5-6x}$$

$$\lim_{x\to-\infty} \frac{\sqrt[2]{9x^2}}{-6x}$$ ← even-even-odd

$$\lim_{x\to-\infty} \frac{3|x^1|}{-6x}$$

$$\lim_{x\to-\infty} \frac{-3\cancel{x}}{-6\cancel{x}} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to-\infty} \frac{2-e^{-4x}}{9+x^2} = \frac{2-\infty}{9+\infty} = \frac{-\infty}{\infty}$$

$$\lim_{x\to-\infty} \frac{-e^{-4x}\cdot -4}{2x}$$

$$\lim_{x\to-\infty} \frac{4e^{-4x}}{2x} = \frac{\infty}{-\infty}$$

$$\lim_{x\to-\infty} \frac{4e^{-4x}\cdot -4}{2} = \frac{4\cdot\infty\cdot -4}{2} = \boxed{-\infty}$$

$$f(x) = \left\{ \begin{array}{ll} \ln(x^2) - x & 0 < x < 1 \\ -\sin\left(\frac{\pi}{2}x\right) & x \geq 1 \end{array} \right\} \quad a = 1$$

Continuous? Differentiable?

1) $f(1) = -\sin\left(\frac{\pi}{2}\right) = -1$

2) $\lim\limits_{x \to 1^-} \ln(x^2) - x = \ln(1) - 1 = 0 - 1 = -1$

$\lim\limits_{x \to 1^+} -\sin\left(\frac{\pi}{2}x\right) = -\sin(\pi/2) = -1$

$\lim\limits_{x \to 1} f(x) = -1$

Know

A deriv. represents...

3) $f(-1) = \lim\limits_{x \to 1} f(x)$

Yes, f is continuous

4) $f'(1)^- = \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} - 1$

$= \frac{2}{x} - 1 = 2 \cdot 1 = 1$

$f'(1)^+ = -\cos\left(\frac{\pi}{2}x\right) \cdot \frac{\pi}{2}$

$= 0 \cdot \pi/2 = 0$

$f'(1)^+ \neq f'(1)^-$

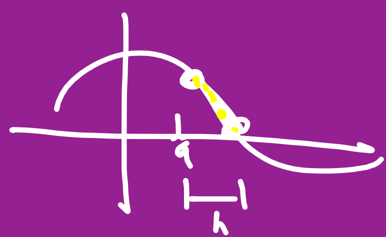
not differentiable

1st Def. of Deriv

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

2nd Def. of Deriv

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



$$f(x)=\frac{x}{x+1} \qquad f(2) \qquad f(x+h)$$

$$\lim_{h\to 0} \frac{\frac{(x+1)}{(x+1)}\cdot\frac{x+h}{x+h+1}-\frac{x}{x+1}\cdot\frac{(x+h+1)}{(x+h+1)}}{h}$$

$$\lim_{h\to 0} \frac{\frac{\cancel{x^2}+\cancel{xh}+\cancel{x}+h-\cancel{x^2}-\cancel{xh}-\cancel{x}}{(x+h+1)(x+1)}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{(x+\underset{0}{h}+1)(x+1)}\cdot\frac{1}{\cancel{h}}$$

$$=\frac{1}{(x+1)^2}$$

$\frac{d}{dx}\sin x = \cos x$ $\quad\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$ $\quad\frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx} e^x = e^x$ $\quad\frac{d}{dx}\ln x = \frac{1}{x}$ $\quad\frac{d}{da} a^x = \ln a \cdot a^x$

$f(x) = 8^{x^2}$

$f'(x) = \ln 8 \cdot 8^{x^2} \cdot 2x$

$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1}$

$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

$f(x) = x^{3x^2}$

$= e^{\ln x^{3x^2}}$

$= e^{3x^2 \cdot \ln x}$

$f'(x) = e^{3x^2 \ln x} \cdot \left[3\cancel{x^2}^{\,1} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 6x\right]$

$= x^{3x^2} \cdot 3x^1 \left[1 + 2\ln x\right]$

$= 3x^{3x^2+1}\left[1 + 2\ln x\right]$