December 15, 2025

Graph Theory

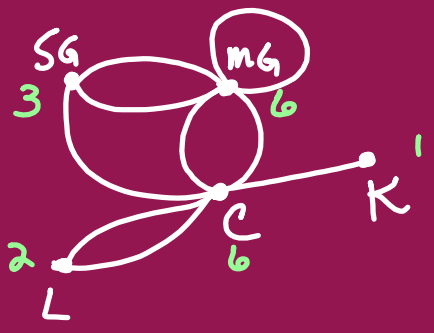
Vertex – Intersection pt.

Edge - Lines that connect vertices

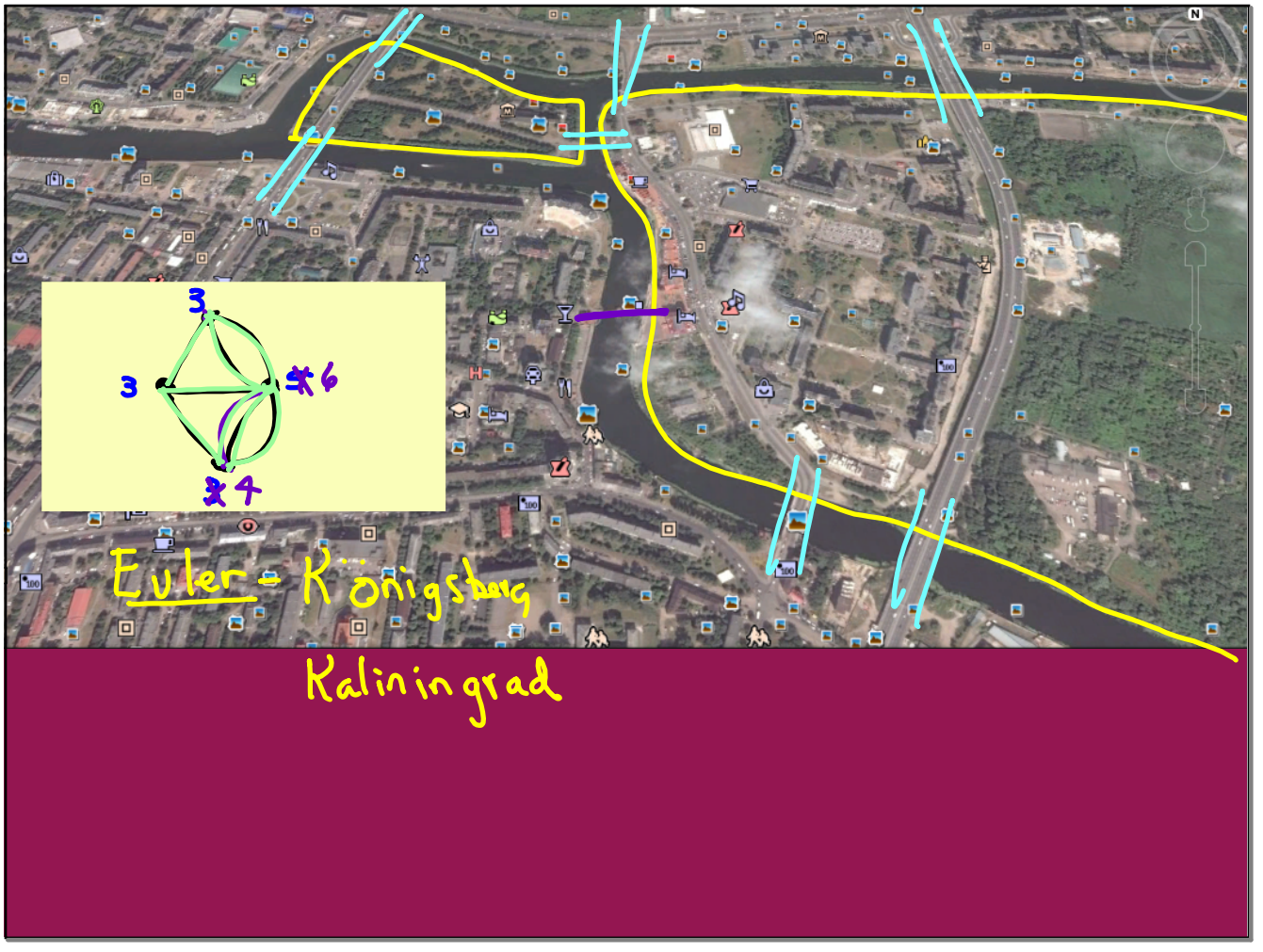
Degree of a vertex = # of edges connected to the vertex

Parallel edges - connect the same 2 vertices

Edges can only cross at a vertex!



December 15, 2025



December 15, 2025

Paths + Circuits

Euler Path

* cross every edge once
* different start + end
* must start + end at odd vertices

Euler circuit

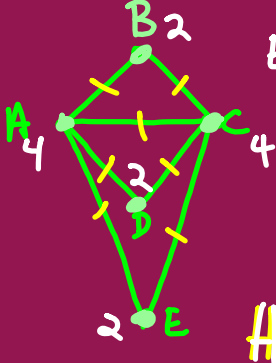
* cross every edge once
* Same start + end point
* all vertices must have even degrees

Hamilton Path

* pass through every vertex once
* different start/end
* no known method to determine when possible

Hamilton Circuit

* pass through every vertex once
* Same start + end point



Euler path - no

Euler circuit - yes

A - B - C - D - A - C - E - A

Ham. path

B - A - E - C - D

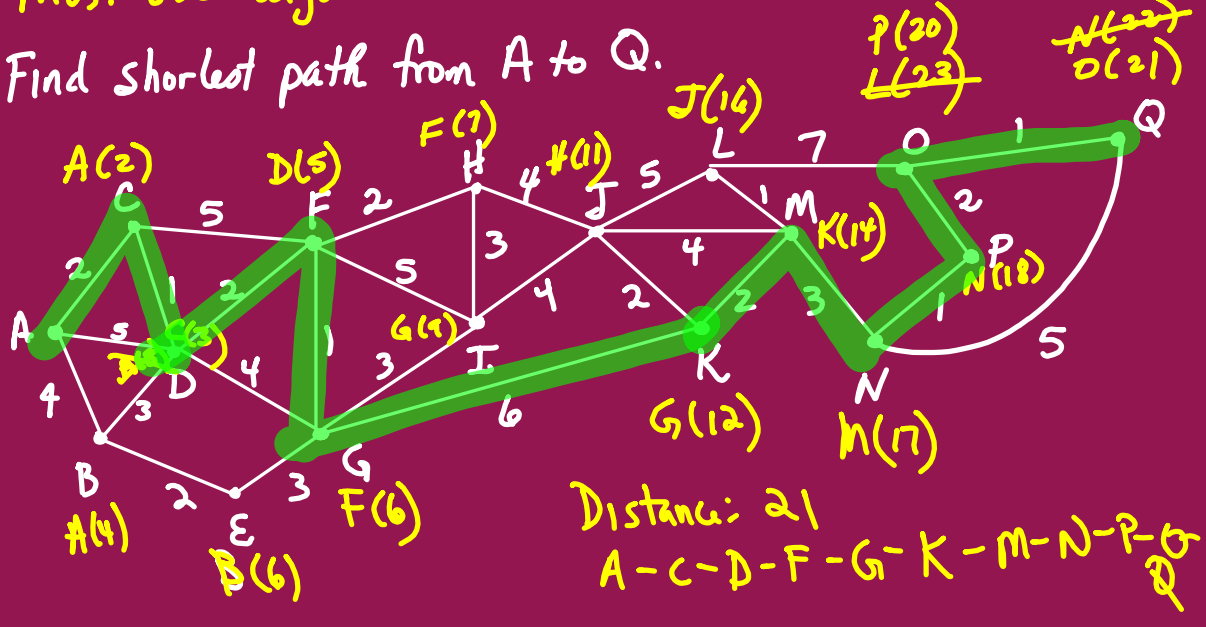
Ham circuit - No

December 15, 2025

Shortest Paths

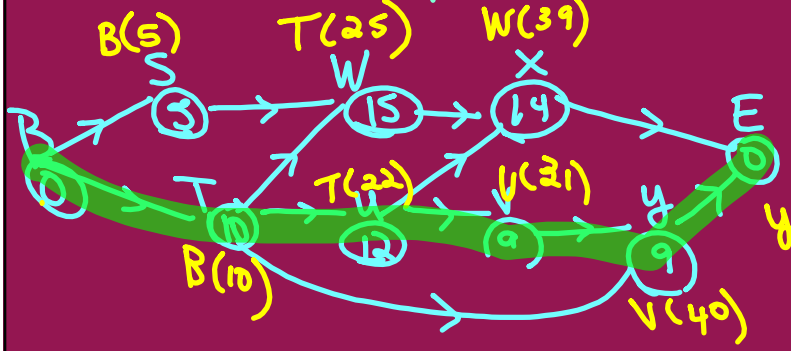
* Weighted graph – edges have a numerical value
* Must use algorithm to find shortest path

Find shortest path from A to Q.



CRITICAL PATHS

* directed graph
* the longest path between 2 vertices
* try to find the minimum time to complete a task



40 min

B-T-U-V-y-E

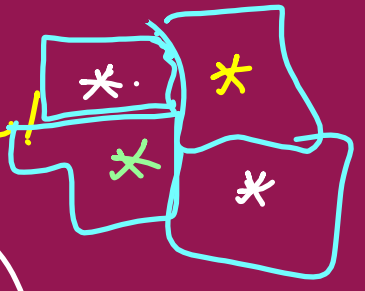
- Roof
  - Ext. - 3
  - Int. - 7

Manufacturing a Smart Watch

Task	Time Required	Prerequisite Tasks
S	5 min	none
T	10	none
U	12	T
V	9	U
W	15	T, S
X	14	U, W
y	9	T, V

Coloring Vertices

Key: Connect the vertices you are actually trying to separate!



Chemical	Cannot be stored with
1	2, 5, 7
2	1, 3, 5
3	2, 4
4	3, 7
5	1, 2, 6, 7
6	5
7	1, 4, 5

Cab 1: 1, 3, 6
Cab 2: 4
Cab 3: 2, 5, 7

December 15, 2025